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RANDOM SEISMIC RESPONSE ANALYSIS OF ADJACENT BUILDINGS COUPLED WITH NON-LINEAR HYSTERETIC DAMPERS

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Coupling adjacent buildings with supplemental damping devices is a practical and effective approach to mitigating structural seismic response. This paper is intended to develop a method for analyzing the random seismic response of a structural system consisting of two adjacent buildings interconnected by non-linear hysteretic damping devices. By modelling the buildings as multi-degree-of-freedom elastic structures and representing the hysteretic dampers with the versatile Bouc–Wen (BW) differential model, an augmented state differential equation is formulated to describe the vibration of the coupled structural system under non-white random seismic excitation. After dealing with the non-linear hysteretic dampers using stochastic linearization technique, a non-linear algebraic Lyapunov equation is derived from which the system mean square response is iteratively solved. The developed method is applicable to the structural system with an arbitrary number of storeys and with connecting dampers at arbitrary storeys. The results of the analysis demonstrate that non-linear hysteretic dampers are effective even if they are placed on a few floor levels. In particular, this type of damping device offers a wideband vibration suppression for earthquake attacks with either low- or high-dominant exciting frequencies. Parametric studies also show that optimum damper parameters and numbers exist which minimize the random seismic response.

1. INTRODUCTION

Interconnecting adjacent structures with passive, active or semi-active control devices to reduce the seismic or wind response of the structures has been an active research subject in recent years. Westermo [1] studied the dynamic implications of connecting closely neighbouring structures by a hinged beam system for the purpose of preventing pounding during earthquakes. Gurley *et al.* [2] studied a system consisting of two adjacent buildings coupled through a single force link for wind-induced vibration control. Kageyama *et al.* [3] proposed to reduce the seismic response of a double-frame building by connecting the inner and outer structures with dampers. Iwanami *et al.* [4] studied the optimum damping and stiffness values of the connecting damper by assuming each of the linked structures as a single-degree-of-freedom system. Luco and De Barros [5] determined the optimal

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distribution of the connecting dampers by modelling the neighbouring buildings as uniform shear beams. Kageyama *et al.* [6] proposed a simplified method to determine the optimum values of the distributed dampers connecting two tower structures of the same height. Sugino *et al.* [7] used the genetic algorithm to optimize the parameters of the connecting dampers. Tamura and Hayashi [8] investigated the earthquake response characteristics of shear walls coupled with vertical dampers.

Seismic protection of adjacent building structures connected with active control devices has been studied by Mitsuta and Seto [9], Luco and Wong [10], Seto [11], Yamada *et al.* [12], Seto [13], Kurihara *et al.* [14], Matsumoto *et al.* [15], Zhang and Xu [16], and Christenson *et al.* [17]. In this context, two or more neighbouring buildings are controlled actively by means of actuators placed between them. Luco and Wong [10] showed that the application of the instantaneous optimal control approach to determine the optimal active control forces between two adjacent structures led to control forces that could be implemented in the form of passive viscous dampers. Yamada *et al.* [12] demonstrated the effectiveness of active seismic control of connected buildings by actuating the joining members to provide negative stiffness. Recently, Christenson *et al.* [17] proposed a semi-active control strategy by using controllable fluid dampers to connect adjacent high-rise buildings for mitigating structural seismic response.

In the reported studies of coupled adjacent structures with passive dampers, the dampers were modelled as linear dampers with constant stiffness and viscous damping values, and consequently linear dynamic problems were addressed. In reality, however, a wide kind of connecting dampers used for this purpose, including those installed in the first building implementing this technology (Kajima Intelligent Building), are the steel elasto-plastic dampers with hysteretic non-linearities [18–20]. The steel elasto-plastic dampers have distinctly different pre-yield and post-yield stiffness, and possess a high capacity of hysteresis damping by plastic deformation. Therefore, it is more reasonable to take into account the non-linear hysteretic behaviour of the connecting dampers in the design analysis of such structural systems. In addition, for the purpose of optimizing design, it is desirable to develop a method that can accurately analyze the seismic response characteristics in consideration of different damper parameters, installation locations and earthquake excitation features.

The present study serves the above purposes. Using the Bouc–Wen differential hysteresis model and the stochastic linearization technique, a random seismic response analysis method for adjacent buildings coupled with non-linear hysteretic dampers under non-white earthquake excitation is developed in this paper. The developed method is suitable for the design evaluation of damper parameters and response control effectiveness because it allows arbitrary degrees of freedom of the structures and arbitrary installation locations of the dampers. As an effort to understand true response characteristics of the non-linear hysteretic dampers coupled structures, parametric studies using the developed method are also conducted for analyzing the effects of the damper parameters, number and distribution and the seismic excitation features on the structural response mitigation capabilities.

2. GOVERNING EQUATION OF MOTION

Figure 1 illustrates a structural system consisting of two adjacent high-rise buildings of N_1 and N_2 ($N_1 \geqslant N_2$) storeys, respectively, connected by non-linear hysteretic dampers at some storeys. It is assumed that with the use of connecting dampers, the two structures remain linearly elastic during earthquakes. The dampers provide passive control force in the horizontal direction. For the shear-type high-rise buildings, the equations of motion of the

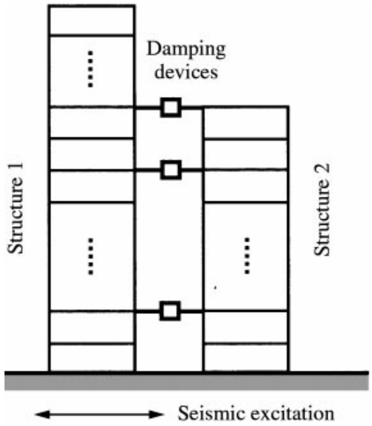

Figure 1. A pair of adjacent buildings interconnected by hysteretic dampers.

coupled structural system can be expressed as

$$\mathbf{M}_1\ddot{\mathbf{x}}_1(t) + \mathbf{C}_1\dot{\mathbf{x}}_1(t) + \mathbf{K}_1\mathbf{x}_1(t) + \mathbf{R}_1(\mathbf{x}_1 - \bar{\mathbf{x}}_2) = \mathbf{E}_1(t), \tag{1}$$

$$\mathbf{M}_2\ddot{\mathbf{x}}_2(t) + \mathbf{C}_2\dot{\mathbf{x}}_2(t) + \mathbf{K}_2\mathbf{x}_2(t) + \mathbf{R}_2(\mathbf{x}_2 - \bar{\mathbf{x}}_1) = \mathbf{E}_2(t), \tag{2}$$

where $\mathbf{M}_i$, $\mathbf{C}_i$ and $\mathbf{K}_i$ $(i = 1, 2)$ are the $N_i \times N_i$-dimensional mass, damping and stiffness matrices of the structure i, respectively, $\mathbf{x}_i$ $(i = 1, 2)$ is the N_i-dimensional lateral displacement vector, $\bar{\mathbf{x}}_1$ the N_2-dimensional sub-vector of $\mathbf{x}_1$ corresponding to $\mathbf{x}_2$ and $\bar{\mathbf{x}}_2$ the N_1-dimensional extended vector of $\mathbf{x}_2$ by adding zero entries, $\mathbf{E}_i$ $(i = 1, 2)$ the N_i-dimensional exciting force vector and $\mathbf{R}_i$ $(i = 1, 2)$ is the hysteretic damping force vector which is determined by

$$\mathbf{R}(\mathbf{x}) = \mathbf{\Lambda}_1\mathbf{x} + \mathbf{\Lambda}_2\mathbf{z}, \qquad \dot{\mathbf{z}}(\mathbf{x}) = \mathbf{G}(\mathbf{z}, \dot{\mathbf{x}})\dot{\mathbf{x}}, \tag{3, 4}$$

in which $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ are diagonal matrices with constant diagonal elements λ_{1j} and λ_{2j} which represent the coefficients of elastic and inelastic components of the hysteretic damping force, $\mathbf{z}$ is a non-dimensional auxiliary argument vector. The hysteretic function $\mathbf{G}$ is a diagonal matrix. When the versatile Bouc–Wen hysteresis model is adopted to represent the connecting dampers, the diagonal element g_j of the matrix $\mathbf{G}$ can be expressed as [21, 22]

$$g_j = \alpha_j - \beta_j|z_j|^{n_j} - \gamma_j \operatorname{sgn}(\dot{x}_j)z_j|z_j|^{n_j-1}, \tag{5}$$

where α_j, β_j, γ_j and n_j are hysteresis model parameters. It has been shown [23] that through appropriate choices of the parameters α_j, β_j, γ_j and n_j, the Bouc–Wen model can represent a wide variety of softening or hardening, smoothly varying or nearly bilinear hysteretic behaviour. As a versatile model, it is apt to offer a satisfactory representation of actual, measured hysteresis loops when the model parameters are properly selected or obtained by identification.

If the connecting dampers possess the initial yield force F_{yj} and yield displacement u_{yj} and have the post-to-pre-yield stiffness ratio μ_j, equation (5) can be expressed in the following non-dimensional form:

$$u_{yj}g_j = \bar{\alpha}_j - \bar{\beta}_j|z_j|^{n_j} - \bar{\gamma}_j \operatorname{sgn}(\dot{x}_j)z_j|z_j|^{n_j-1}, \tag{6}$$

where the dimensionless parameters are $\bar{\alpha}_j = \alpha_j u_{yj}$, $\bar{\beta}_j = \beta_j u_{yj}$ and $\bar{\gamma}_j = \gamma_j u_{yj}$. When m_{dj} connecting dampers are installed at the jth floor, the diagonal elements of the matrices $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ are, respectively, $\lambda_{1j} = m_{dj}\mu_j F_{yj}/u_{yj}$ and $\lambda_{2j} = m_{dj}(1-\mu_j)F_{yj}$. It should be noted that the vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ have the same magnitudes but opposite directions.

In the case of seismic excitation, the vector $\mathbf{x}_i$ $(i = 1, 2)$ should be regarded as a relative displacement response of the structural system with respect to the ground, and the excitation force vectors are expressed as

$$\mathbf{E}_1 = -\ddot{x}_g(t)\mathbf{M}_1\tilde{\mathbf{I}}_1, \quad \mathbf{E}_2 = -\ddot{x}_g(t)\mathbf{M}_2\tilde{\mathbf{I}}_2, \tag{7a, b}$$

where $\ddot{x}_g(t)$ is the ground acceleration excitation and $\tilde{\mathbf{I}}_i$ $(i = 1, 2)$ the N_i-dimensional unit vector.

3. PRESENTATION OF ANALYSIS METHOD

3.1. STOCHASTIC LINEARIZATION TREATMENT

Since the earthquake excitation is indeterministic and stochastic in nature, the dynamic response of the coupled structural system under random seismic excitation is analyzed in this study. The stochastic linearization technique [24, 25] is first utilized to deal with the hysteretic non-linearities of the connecting dampers. By minimizing the total mean square error, the hysteretic constitutive relation (4) can be replaced by the following linearized equation:

$$\dot{\mathbf{z}} + \mathbf{C}_e\dot{\mathbf{x}} + \mathbf{K}_e\mathbf{z} = \mathbf{0}, \tag{8}$$

where $\mathbf{C}_e$ and $\mathbf{K}_e$ are diagonal matrices with the diagonal elements c_{ej} and k_{ej} respectively. For the Bouc–Wen hysteresis model, we have

$$c_{ej} = -E\left[\frac{\partial(g_j\dot{x}_j)}{\partial\dot{x}_j}\right] = -\alpha_j + \beta_j E[|z_j|^{n_j}] + \gamma_j E\left[z_j|z_j|^{n_j-1}\frac{\partial|\dot{x}_j|}{\partial\dot{x}_j}\right], \tag{9a}$$

$$k_{ej} = -E\left[\dot{x}_j\frac{\partial g_j}{\partial z_j}\right]$$

$$= n_j\beta_j E\left[\dot{x}_j|z_j|^{n_j-1}\frac{\partial|z_j|}{\partial z_j}\right] + \gamma_j E\,[|\dot{x}_j||z_j|^{n_j-1}] + (n_j-1)\gamma_j E\left[|\dot{x}_j|z_j|z_j|^{n_j-2}\frac{\partial|z_j|}{\partial z_j}\right]. \tag{9b}$$

Here $E[\cdot]$ denotes the expectation manipulation.

If the random excitation E_i $(i = 1, 2)$ is a Gaussian process, the responses $\mathbf{x}_i$ $(i = 1, 2)$ and the hysteretic forces $\mathbf{z}_i$ $(i = 1, 2)$ in the equivalently linearized system are jointly Gaussian. Thus, the equivalent parameters $\mathbf{C}_e$ and $\mathbf{K}_e$ can be evaluated in terms of the second moments of $\dot{\mathbf{x}}_i$ and $\mathbf{z}_i$ $(i = 1, 2)$. It is obtained in the case $n_j = 1$ that

$$c_{ej} = -\alpha_j + \beta_j\sqrt{\frac{2}{\pi}E[z_j^2]} + \gamma_j E[\dot{x}_j z_j]\sqrt{\frac{\pi}{2E[\dot{x}_j^2]}}, \tag{10a}$$

$$k_{ej} = \gamma_j\sqrt{\frac{2}{\pi}E[\dot{x}_j^2]} + \beta_j E[\dot{x}_j z_j]\sqrt{\frac{\pi}{2E[z_j^2]}}. \tag{10b}$$

The expressions of the equivalent parameters for the case $n_j \neq 1$ have been given by Wen [24]. If the random excitation is stationary, the parameter matrices $\mathbf{C}_e$ and $\mathbf{K}_e$ are time invariant and consequently, the random dynamic response is stationary. It should be noted that because of the use of different estimate approaches, expressions (10a) and (10b) derived here are slightly different from those given in reference [24].

By combining equations (1)–(3) and (8), the equation of motion of the coupled structural system can be rewritten in the following first order differential equation form:

$$\dot{\mathbf{Y}} = \mathbf{A}\mathbf{Y} + \mathbf{F}(t), \tag{11}$$

where

$$\mathbf{Y} = \{\mathbf{x}_1^{\mathrm{T}}\ \dot{\mathbf{x}}_1^{\mathrm{T}}\ \mathbf{z}^{\mathrm{T}}\ \mathbf{x}_2^{\mathrm{T}}\ \dot{\mathbf{x}}_2^{\mathrm{T}}\}^{\mathrm{T}}, \tag{12a}$$

$$\mathbf{F} = \{\mathbf{0}^{\mathrm{T}}\ \mathbf{E}_1^{\mathrm{T}}\ \mathbf{M}_1^{-\mathrm{T}}\ \mathbf{0}^{\mathrm{T}}\ \mathbf{0}^{\mathrm{T}}\ \mathbf{E}_2^{\mathrm{T}}\ \mathbf{M}_2^{-\mathrm{T}}\}^{\mathrm{T}}, \tag{12b}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{M}_1^{-1}(\mathbf{K}_1 + \mathbf{\Lambda}'_{1a}) & -\mathbf{M}_1^{-1}\mathbf{C}_1 & \mathbf{M}_1^{-1}\mathbf{\Lambda}'_2 & \mathbf{M}_1^{-1}\mathbf{\Lambda}''_{1a} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}'_e & -\mathbf{K}_e & \mathbf{0} & -\mathbf{C}''_e \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_2 \\ \mathbf{M}_2^{-1}\mathbf{\Lambda}'_{1b} & \mathbf{0} & -\mathbf{M}_2^{-1}\mathbf{\Lambda}''_2 & -\mathbf{M}_2^{-1}(\mathbf{K}_2 + \mathbf{\Lambda}''_{1b}) & -\mathbf{M}_2^{-1}\mathbf{C}_2 \end{bmatrix}, \tag{12c}$$

in which $\mathbf{z}$ is the N_3-dimensional hysteretic variable vector where $N_3(\leqslant N_2 \leqslant N_1)$ is the number of floors at which the connecting dampers are installed, $\mathbf{I}_1$ and $\mathbf{I}_2$ are the $N_1 \times N_1$ and $N_2 \times N_2$ identity matrices, $\mathbf{\Lambda}'_2$ and $\mathbf{\Lambda}''_2$ the $N_1 \times N_3$ and $N_2 \times N_3$ matrices containing elements λ_{2j} and zeros, $\mathbf{\Lambda}'_{1a}$, $\mathbf{\Lambda}''_{1a}$, $\mathbf{\Lambda}'_{1b}$ and $\mathbf{\Lambda}''_{1b}$ the $N_1 \times N_1$, $N_1 \times N_2$, $N_2 \times N_1$ and $N_2 \times N_2$ matrices containing elements λ_{1j} and zeros, $\mathbf{C}'_e$ and $\mathbf{C}''_e$ the $N_3 \times N_1$ and $N_3 \times N_2$ extended matrices of $\mathbf{C}_e$ by adding zero elements.

3.2. AUGMENTED STATE EQUATION UNDER NON-WHITE EXCITATION

The earthquake excitation properties including the dominant frequency and damping of ground excitation (site soil) may affect the seismic response reduction effectiveness of the coupled structural system. In the present study, the random seismic excitation is modelled as a non-white stationary Gaussian process, and the excitation power spectrum density is represented by the Kanai–Tajimi spectrum [26, 27]:

$$\Phi(\omega) = \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2} S_0. \tag{13}$$

The Kanai–Tajimi spectrum can be obtained from a second order differential system subjected to white-noise excitation. In this way, the ground acceleration excitation $\ddot{x}_g(t)$ is produced by

$$\ddot{x}_g(t) = -\,\omega_g^2 y_g - 2\zeta_g\omega_g\dot{y}_g, \qquad \ddot{y}_g + 2\zeta_g\omega_g\dot{y}_g + \omega_g^2 y_g = \xi(t), \tag{14a, b}$$

where $\xi(t)$ is the stationary Gaussian white noise with intensity $D = 2\pi S_0$, y_g the response of the filtering system, ω_g and ζ_g are the natural frequency and damping ratio of the filter, which represent the characteristics of site soil.

Since equation (14) is a linear filtering system, it can be incorporated into equation (11) to form the following augmented first order differential equation:

$$\dot{\tilde{\mathbf{Y}}} = \tilde{\mathbf{A}}\tilde{\mathbf{Y}} + \tilde{\mathbf{F}}(t), \tag{15}$$

in which

$$\tilde{\mathbf{Y}} = \{\mathbf{x}_1^{\mathrm{T}}\ \dot{\mathbf{x}}_1^{\mathrm{T}}\ \mathbf{z}^{\mathrm{T}}\ \mathbf{x}_2^{\mathrm{T}}\ \dot{\mathbf{x}}_2^{\mathrm{T}}\ y_g\ \dot{y}_g\}^{\mathrm{T}}, \tag{16a}$$

$$\tilde{\mathbf{F}} = \{\mathbf{0}^{\mathrm{T}}\ \mathbf{0}^{\mathrm{T}}\ \mathbf{0}^{\mathrm{T}}\ \mathbf{0}^{\mathrm{T}}\ \mathbf{0}^{\mathrm{T}}\ 0\ \xi(t)\}^{\mathrm{T}}, \tag{16b}$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{M}_1^{-1}(\mathbf{K}_1 + \mathbf{\Lambda}'_{1a}) & -\mathbf{M}_1^{-1}\mathbf{C}_1 & \mathbf{M}_1^{-1}\mathbf{\Lambda}'_2 & \mathbf{M}_1^{-1}\mathbf{\Lambda}''_{1a} & \mathbf{0} & \omega_g^2\tilde{\mathbf{I}}_1 & 2\zeta_g\omega_g\tilde{\mathbf{I}}_1 \\ \mathbf{0} & \mathbf{C}'_e & -\mathbf{K}_e & \mathbf{0} & -\mathbf{C}''_e & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_2^{-1}\mathbf{\Lambda}'_{1b} & \mathbf{0} & -\mathbf{M}_2^{-1}\mathbf{\Lambda}''_2 & -\mathbf{M}_2^{-1}(\mathbf{K}_2 + \mathbf{\Lambda}''_{1b}) & -\mathbf{M}_2^{-1}\mathbf{C}_2 & \omega_g^2\tilde{\mathbf{I}}_2 & 2\zeta_g\omega_g\tilde{\mathbf{I}}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\omega_g^2 & -2\zeta_g\omega_g \end{bmatrix}. \tag{16c}$$

3.3. SOLUTION OF RANDOM STATIONARY RESPONSE

Equation (15) is a linearized time-invariant system subjected to stationary white-noise excitation. The statistical response of this system can be readily obtained in an integration form. However, because the coefficient matrix $\tilde{\mathbf{A}}$ depends on the second moments of the response $\tilde{\mathbf{Y}}$, an iterative solution procedure is required. The response of the augmented system (15) can be expressed as

$$\tilde{\mathbf{Y}}(t) = \mathrm{e}^{\tilde{\mathbf{A}}(t-t_0)}\tilde{\mathbf{Y}}_0 + \int_{t_0}^{t} \mathrm{e}^{\tilde{\mathbf{A}}(t-\tau)}\tilde{\mathbf{F}}(\tau)\,\mathrm{d}\tau \tag{17}$$

and therefore the mean response and correlation function are obtained, respectively, as

$$E[\tilde{\mathbf{Y}}] = \mathrm{e}^{\tilde{\mathbf{A}}(t-t_0)}E[\tilde{\mathbf{Y}}_0] + \int_{t_0}^{t} \mathrm{e}^{\tilde{\mathbf{A}}(t-\tau)}E[\tilde{\mathbf{F}}(\tau)\,\mathrm{d}\tau, \tag{18}$$

$$E[\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^{\mathrm{T}}] = \mathrm{e}^{\tilde{\mathbf{A}}(t-t_0)}E[\tilde{\mathbf{Y}}_0\tilde{\mathbf{Y}}_0^{\mathrm{T}}]\mathrm{e}^{\tilde{\mathbf{A}}^{\mathrm{T}}(t_2-t_0)} + \int_{t_0}^{\min(t_1,\,t_2)} \mathrm{e}^{\tilde{\mathbf{A}}(t_1-\tau)}\mathbf{D}_F\mathrm{e}^{\tilde{\mathbf{A}}^{\mathrm{T}}(t_2-\tau)}\,\mathrm{d}\tau, \tag{19}$$

where t_0 is the initial time, $\tilde{\mathbf{Y}}_0$ the initial state and $\mathbf{D}_F$ the intensity matrix of the white-noise excitation vector $\tilde{\mathbf{F}}$.

The correlation function matrix $\mathbf{W} = E[\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^{\mathrm{T}}]$ should satisfy the following differential equation which is derived from equations (15) and (17). That is,

$$\begin{aligned} \dot{\mathbf{W}}(t) &= E[\dot{\tilde{\mathbf{Y}}}(t)\tilde{\mathbf{Y}}^{\mathrm{T}}(t)] + E[\tilde{\mathbf{Y}}(t)\dot{\tilde{\mathbf{Y}}}^{\mathrm{T}}(t)] \\ &= \tilde{\mathbf{A}}\mathbf{W}(t) + \mathbf{W}(t)\tilde{\mathbf{A}}^{\mathrm{T}} + \mathbf{D}_F. \end{aligned} \tag{20}$$

Equation (20) is referred to as a differential Lyapunov equation. For the stationary response of the system under stationary random excitation, it reduces to the following algebraic Lyapunov equation:

$$\tilde{\mathbf{A}}\mathbf{W} + \mathbf{W}\tilde{\mathbf{A}}^{\mathrm{T}} + \mathbf{D}_F = \mathbf{0}. \tag{21}$$

The correlation function matrix and then the mean square response can be determined by solving equation (21). Since the coefficient matrix $\tilde{\mathbf{A}}$ is dependent on $\mathbf{W}$, equation (21) is a non-linear algebraic equation and should be solved by numerical iteration. In the present study, equation (21) with respect to the square matrix $\mathbf{W}$ is rearranged into that with respect to a one-dimensional vector $\{\bar{\mathbf{W}}\}$, so that the first two terms on the left-hand side of equation (21) can be combined. In recognizing that $\mathbf{W}$ is a symmetric matrix, the vector $\{\bar{\mathbf{W}}\}$ only consists of the upper triangular elements of the matrix $\mathbf{W}$. The resulting non-linear equation with respect to $\{\bar{\mathbf{W}}\}$ can be expressed in an iterative form as follows:

$$[\bar{\tilde{\mathbf{A}}}(\bar{\mathbf{W}}^{(k)})]\,\{\bar{\mathbf{W}}^{(k+1)}\} + \{\bar{\mathbf{D}}_F\} = \mathbf{0}. \tag{22}$$

Equation (22) is a system of linearized algebraic equations at each iterative step, and can be readily solved. By taking the correlation function of the corresponding linear system as the initial guess, the mean square response of the coupled structural system is obtained by iteratively solving equation (22) until $\|\bar{\mathbf{W}}^{(k+1)} - \bar{\mathbf{W}}^{(k)}\| / \|\bar{\mathbf{W}}^{(k)}\| \to 0$. Usually, over 15 iterations are needed to reach a converged solution for the given tolerance 10^{-5}. In the parametric studies, a sweeping approach is used. When doing this, the response obtained for a specific parameter value is taken as the initial estimate for the iterative solution for the next nearby parameter value. This parameter-sweeping approach can accelerate the iterative solution and ensure convergence so long as the sweeping step is small enough.

4. PARAMETRIC STUDIES

Using the developed method, parametric studies are conducted with respect to a structural system which consists of a 20-storey building and a 10-storey building coupled with non-linear hysteretic dampers at a few floors. The mass of each floor is $m = 1{\cdot}6 \times 10^6$ kg, the interstorey stiffness is $k = 1{\cdot}2 \times 10^{10}$ N/m, and the interstorey viscous damping coefficient is $c = 2{\cdot}4 \times 10^8$ N s/m. The first six natural frequencies of the 20-storey building are 1·06, 3·16, 5·25, 7·30, 9·32 and 11·28 Hz, and the first four natural frequencies of the 10-storey building are 2·06, 6·13, 10·07 and 13·78 Hz.

The dimensionless hysteresis model parameters of the connecting dampers are taken as $\bar{\alpha} = 1{\cdot}0$, $\bar{\beta} = 0{\cdot}3$, $\bar{\gamma} = 0{\cdot}1$, $n = 1{\cdot}0$ and $\mu = 0{\cdot}1$. The initial yield displacement and force of the dampers are taken to be $u_y = 0{\cdot}004$ m and $F_y = 150$ kN unless otherwise indicated. The hysteresis loops of the connecting dampers governed by these model parameters are illustrated in Figure 2. The spectral parameters of the random seismic excitation are taken to be $\zeta_g = 0{\cdot}3$, $\omega_g = 23{\cdot}0$ rad/s and $D = 0{\cdot}4$ m^2/s^3 unless otherwise indicated.

4.1. CONNECTING DAMPERS AT SINGLE STOREY

The response analysis is first performed for the case where the connecting dampers are installed only at one storey. Figures 3–5 show the response prediction results when 10 hysteretic dampers are installed between the buildings at the 10th floor level. Figure 3 shows the root mean square (RMS) displacement responses of the uncoupled and coupled

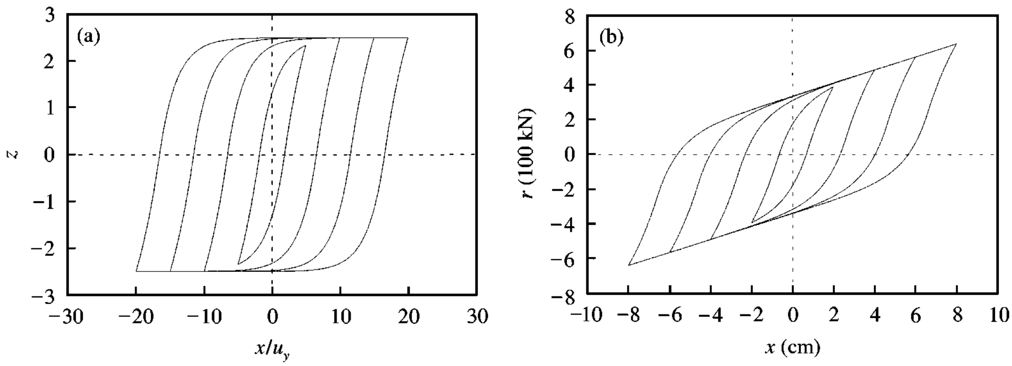

Figure 2. Hysteresis loops of connecting dampers: (a) z versus x/u_y; (b) r versus x.

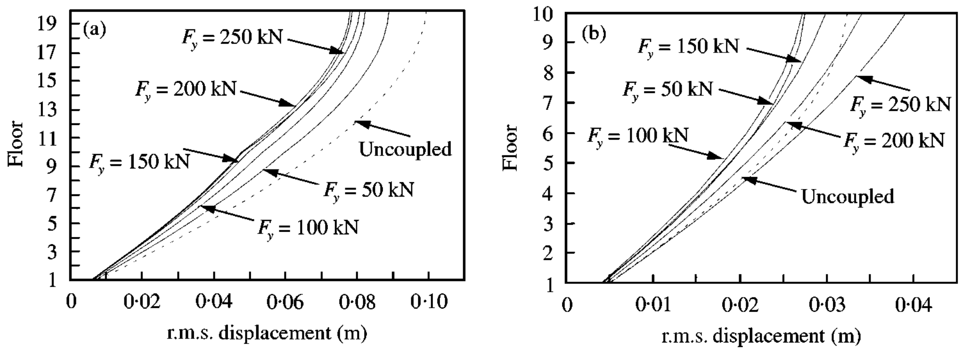

Figure 3. Displacement responses for different F_y values (dampers at 10th floor): (a) 20-storey building; (b) 10-storey building.

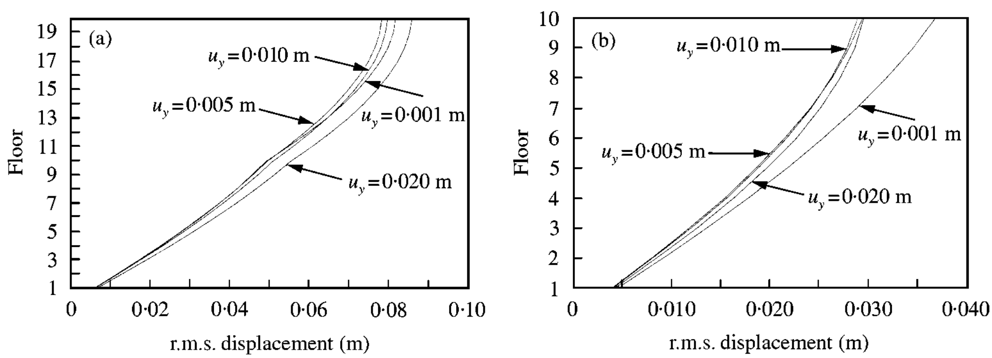

Figure 4. Displacement responses for different u_y values (dampers at 10th floor): (a) 20-storey building; (b) 10-storey building.

buildings when the initial yield force (F_y) of the dampers equals 50, 100, 150, 200 and 250 kN respectively. It is seen that $F_y = 150$ kN ($F_y/u_y = 3{\cdot}75 \times 10^4$ kN/m) is preferable for mitigating the structural responses of both the buildings. In this situation, the maximum r.m.s. displacement response is reduced by 20·8% in comparison with the uncoupled structures. Figure 4 shows the r.m.s. displacement responses of the coupled buildings when

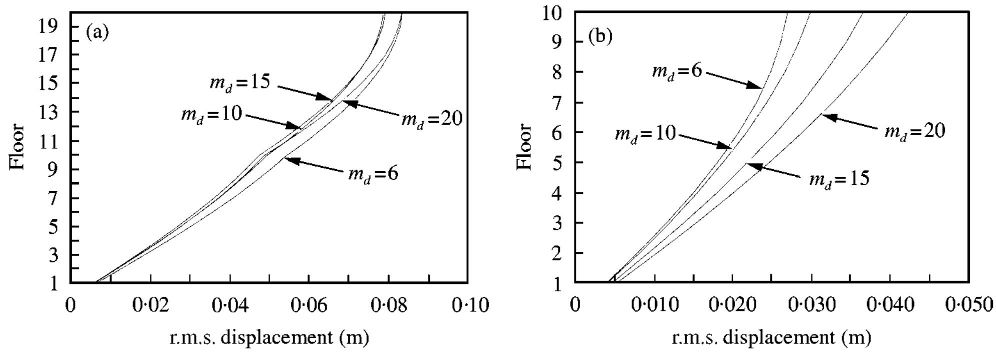

Figure 5. Displacement responses for different m_d values (dampers at 10th floor): (a) 20-storey building; (b) 10-storey building.

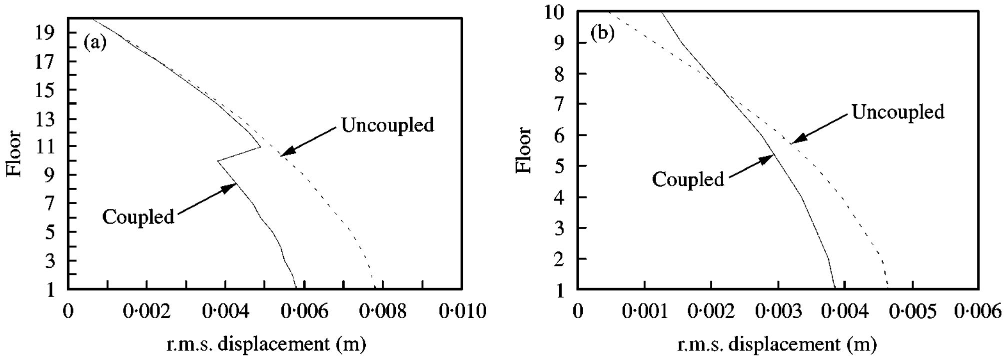

Figure 6. Interstorey drifts of coupled and uncoupled buildings: (a) 20-storey building; (b) 10-storey building.

the initial yield displacement (u_y) of the dampers is taken as 0·001, 0·005, 0·01 and 0·02 m respectively. It is observed that u_y being equal to 0·005 m is optimal for the structural response mitigation of both the buildings. Figure 5 shows the r.m.s. displacement responses of the coupled buildings when changing the number (m_d) of dampers installed at the 10th floor. The number of installed dampers is selected to be 6, 10, 15 and 20 respectively. It is seen that there is an optimal option for the number of dampers (10–15) to achieve maximum response reduction.

Figure 6 shows the r.m.s. interstorey drifts of the coupled and uncoupled buildings. For the taller structure (20-storey building), the interstorey drifts are significantly reduced at the storeys beneath the floor (10th floor) installed with the dampers. A 25·6% reduction over the uncoupled response of the maximum r.m.s interstorey drift is observed. For the shorter structure (10-storey building), however, the interstorey drifts are adversely raised at the storeys near the damper floor. Figure 7 shows the r.m.s. absolute acceleration responses of the coupled and uncoupled buildings. It is observed that the acceleration response reduction is more notable in the shorter structure than in the taller structure. A 19·2% reduction over the uncoupled response of the maximum r.m.s. absolute acceleration is obtained.

The transient dynamic responses of the coupled and uncoupled buildings subject to the El Centro 1940 NS earthquake excitation have also been evaluated. It is found that the maximum peak displacement and acceleration are reduced by 19·5 and 22·2%, respectively,

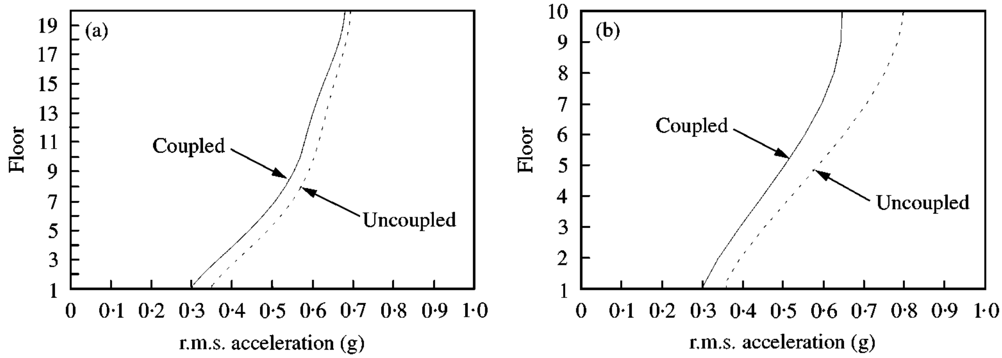

Figure 7. Absolute acceleration responses of coupled and uncoupled buildings: (a) 20-storey building; (b) 10-storey building.

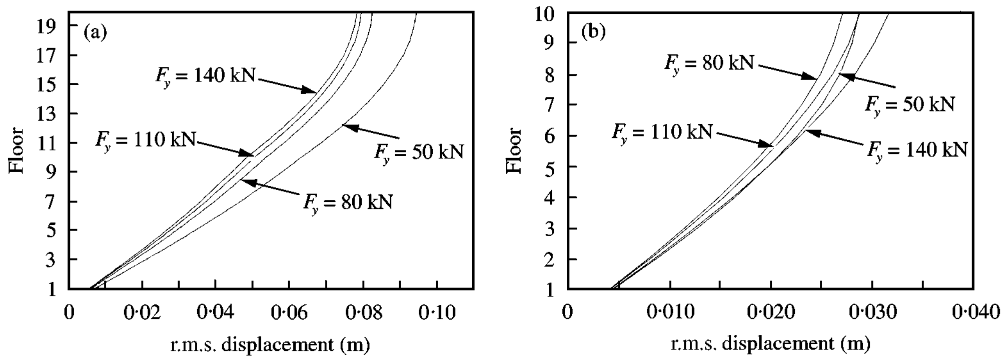

Figure 8. Displacement responses with dampers installed at eighth and 10th floors: (a) 20-storey building; (b) 10-storey building.

being close to the preceding results based on the random analysis. These results also coincide with the response reduction evaluation of the Kajima Intelligent Building Complex interconnected with steel elasto-plastic dampers [18].

4.2. CONNECTING DAMPERS AT SEVERAL STOREYS

The response analysis is then performed with the connecting dampers being installed at two storeys and at three storeys respectively. Figure 8 shows the r.m.s. displacement responses of the coupled buildings when 10 dampers are installed at the 10th floor level and three dampers are installed at the eighth floor level. In this case, taking $F_y = 110$ kN is preferable to reducing the structural responses of both the buildings. Figure 9 shows the r.m.s. displacement responses of the coupled buildings when 10 dampers are installed at the 10th floor level, three dampers at the eighth floor level, and three dampers at the sixth floor level. In this case, $F_y = 80$ kN is the optimal choice. It has been identified above that $F_y = 150$ kN is optimum when the dampers are installed only at the 10th floor. Therefore, it can be concluded that when more dampers are installed, the optimal value of the initial yield force of the dampers is reduced. It is also found that the structural response mitigation capabilities of installing the dampers at one properly selected storey are comparable to those of installing the dampers at multiple storeys.

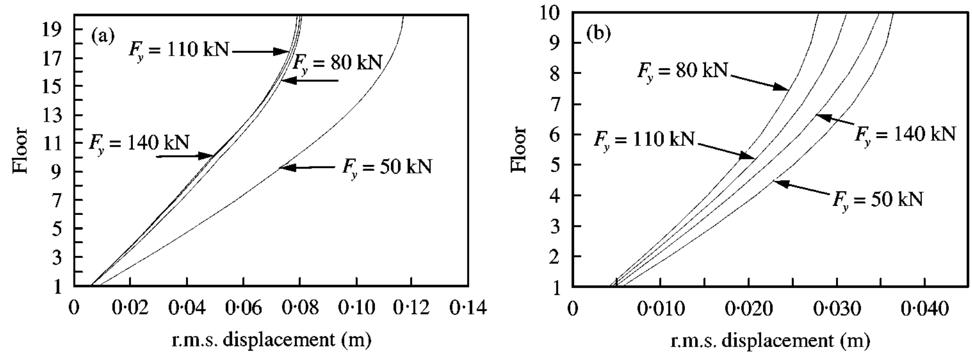

Figure 9. Displacement responses with dampers installed at sixth, eighth and 10th floors: (a) 20-storey building; (b) 10-storey building.

4.3. INFLUENCE OF SEISMIC EXCITATION PROPERTIES

The influence of intensity and dominant frequency of the random seismic excitation on the structural response is eventually evaluated. Figure 10 shows the top-floor r.m.s. displacement responses of the coupled and uncoupled buildings against the spectral intensity (D) of the random seismic excitation. The 10 connecting dampers are only installed at the 10th floor. It is observed that with the enhancement of seismic excitation intensity, the non-linear hysteretic dampers provide increasing structural response mitigation capability. Figure 11 shows the top-floor r.m.s. displacement responses of the coupled and uncoupled buildings against the dominant frequency (ω_g) of the seismic excitation. The earthquake dominant frequency depends on the characteristics of the site soil. An analysis of 47 earthquake records indicated ω_g ranging from 5 to 35 rad/s [28]. It is revealed from Figure 11 that the non-linear hysteretic dampers provide robust seismic response mitigation capabilities for the taller structure (20-storey building) irrespective of the seismic excitation frequency. The non-linear hysteretic dampers are more efficient than the corresponding linear dampers ($\beta = \gamma = 0$) in suppressing random earthquake response as shown in Figures 12 and 13.

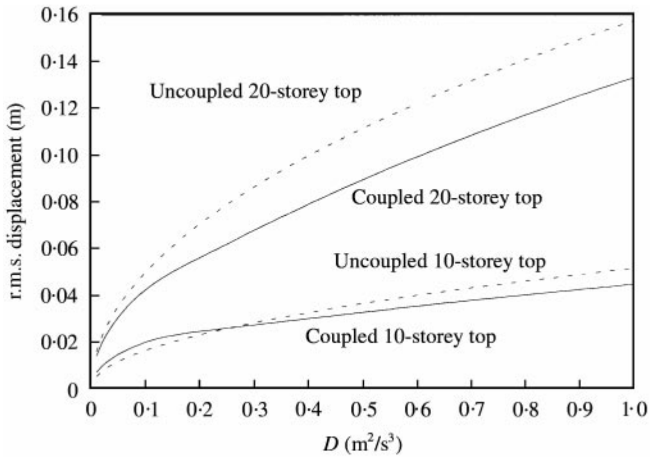

Figure 10. Top-floor responses versus intensity of seismic excitation.

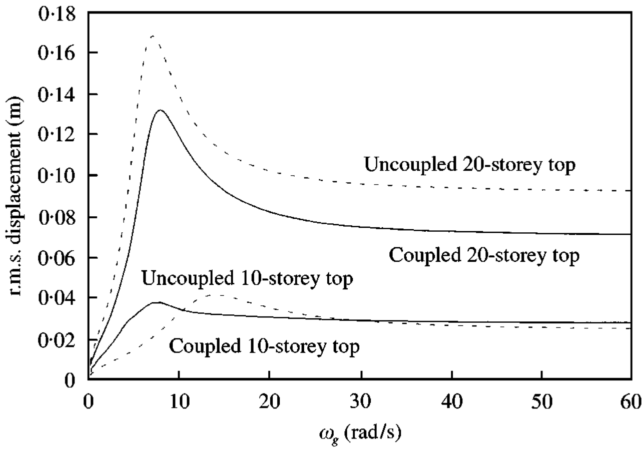

Figure 11. Top-floor responses versus frequency of seismic excitation.

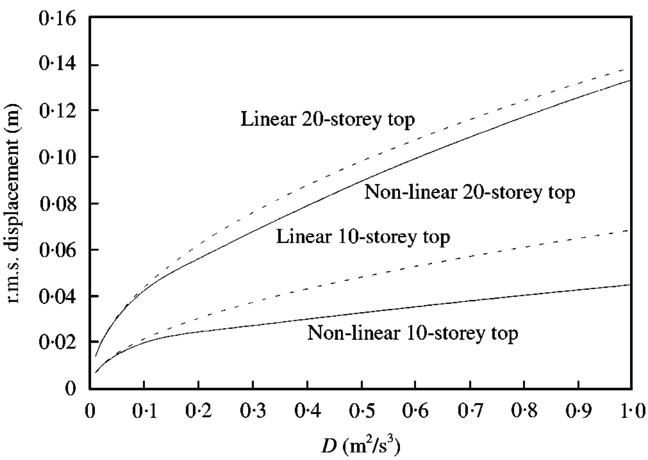

Figure 12. Top-floor responses of coupled buildings versus intensity seismic excitation.

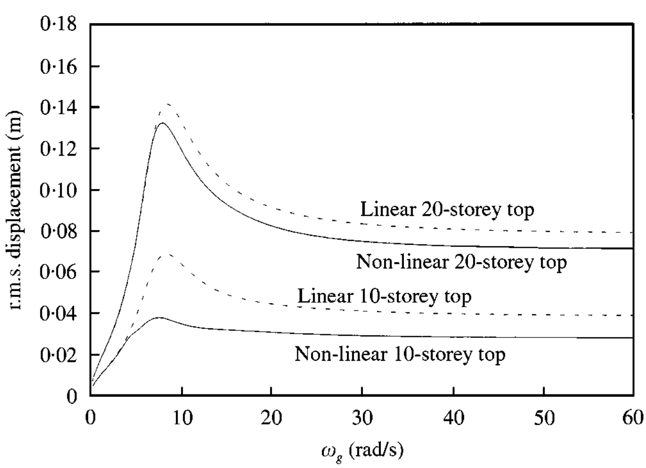

Figure 13. Top-floor responses of coupled buildings versus frequency of seismic excitation.

In the numerical example illustrated, the maximum mean-square response of the taller structure is dominant and is therefore regarded as the most important control parameter. Different response levels of the coupled structural system can be achieved by adjusting the parameter values of connecting hysteretic dampers. For a specific control requirement for each of the adjacent buildings, suitable dampers can be determined based on parameter and optimization analysis.

5. CONCLUSIONS

Coupling two or more adjacent high-rise buildings with supplemental damping devices is a promising method of mitigating structural seismic response. This technique can also be used to reduce the pounding between adjacent buildings during earthquakes. The commonly used dampers for this purpose are the steel elasto-plastic dampers that exhibit hysteretic non-linearities. In the present study, a random seismic response analysis method for the adjacent buildings interconnected with this kind of dampers was developed. This method has the following attributes: (a) It incorporates the non-linear hysteretic model of the connecting dampers; (b) It allows arbitrary degrees of freedom of the structures and arbitrary distribution of the dampers, and is therefore suitable for design analysis to determine optimal damper specification (parameters, number and installation locations) and (c) It takes into account the influence of the seismic excitation (site soil) properties and therefore evaluates the response mitigation capabilities of the coupled structural system more accurately.

Using the developed method, parametric studies were conducted to get an insight into the random seismic response characteristics of the coupled structural system. The following conclusions are drawn based on the parametric studies: (a) The seismic response mitigation of the adjacent buildings can be achieved by installing the connecting dampers at a few floors only; (b) The hysteretic dampers can provide a wideband vibration suppression from the earthquake attack with low- or high-excitation frequencies and (c) There is an optimal choice of the parameters, number and distribution of the hysteretic dampers so as to yield a minimum structural response of the coupled structural system.

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